On the Observability of the Point Kinetic Equations of a Nuclear Reactor

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Abstract

In this work, the conditions to estimate non-measurable states of a nuclear reactor are studied. In a nuclear reactor, the main non-measurable variables are the delayed neutron precursor concentration and the internal reactivity. By certain techniques, these variables can be estimated from the reactor’s input and output, that is the external reactivity and the neutron density, respectively. This reconstruction task is of fundamental importance if a state feedback control scheme or a failure detection system is to be implemented on the reactor. However, to guarantee the correct and safe functioning of these systems, it is necessary first to know under which conditions such reconstruction is possible. To this end, an exhaustive analysis of the observability conditions of the point kinetic equations of a reactor, in particular, a TRIGA-type is carried out here. The analysis consists of two stages. First, the equations are linearized with respect to any possible equilibrium point and not to a particular one as is usually made, thus exploiting completely the possibilities offered by the linearization technique. Then, an analysis of the observability of the general linear model is performed, obtaining the interesting result that the non-measurable states are observable except when the effective precursor radioactive decay constant and the reciprocal of the fuel-to-coolant heat transfer mean time have the same value, which does not occur, in practice, in a TRIGA reactor. Given that the system under study is nonlinear in nature, a follow-up analysis should be carried out to investigate if additional conditions to guarantee the observability exist. Thus, the second stage consists on extending the observability analysis to the nonlinear system. To better understand this second analysis, some results of the observability theory of nonlinear systems are summarized. The relevant result of the second stage is that, contrary to the linear case, the observability of the reactor’s nonlinear model depends on the control signal values. Thus, a restriction is imposed on the system’s input to guarantee the observability of the original point kinetic equations.
1. INTRODUCTION

Due to the absence of adequate sensors, some of the variables associated with the dynamics of a nuclear reactor are not available for measurement. If, however, a state feedback controller or a fault detection system is to be implemented on the reactor, the unmeasurable variables could be required by the control law or the decision making rules. Consequently, an aggregated dynamic system (observation scheme) based on measurable states must be incorporated to reconstruct the unavailable variables. Before attempting the use of any observation scheme, a basic question has to be resolved, namely, What are the conditions under which the reconstruction problem of the reactor’s unmeasurable states has a solution? This question leads to determining the observability characteristics of the reactor’s states. The corresponding observability analysis is carried out on the reactor’s point kinetic equations.

It is worth mentioning that in most cases reported in the literature [1-4], both the observability analysis and the reconstruction or estimation of the non-measurable variables has been realized on linearized reactor’s kinetic equations, where the linearization is made around a specific equilibrium point. Since the dynamic behavior of a nuclear reactor exhibits strong nonlinearities, the results obtained are only satisfactory in a small neighborhood about such equilibrium point.

One of the goals of this work is to extend the results on the observability of the reactor at any power level over a wider operation range, which is equivalent to consider some of the reactor’s nonlinear dynamics. To attain this goal, the point kinetic equations are linearized about any possible equilibrium point, followed by their corresponding observability analysis. Another relevant objective is to obtain the conditions for the observability of the nonlinear model of the reactor. To this end, a second analysis is made using the observability theory of nonlinear systems. As it will be shown, the condition obtained on the input signal for full observability can be used as a restriction on the universe of discourse of the input signal, thus guaranteeing the reconstruction of the non-measurable states and consequently their safe use in control schemes that require full state feedback or in fault detection systems that require all of the system’s variables in their decision rules.

2. THE REACTOR’S POINT KINETIC EQUATIONS

The operation of any nuclear fission reactor is based on the fission of unstable isotopes; generally, the employed isotope is U^{235}. For this isotope, the fission process can be described by

\[ n_1 + U^{235} \to F_1 + F_2 + m \cdot n_2 + \text{Energy (200MeV)} \]  

(1)

where \( n_1 \) is the thermal neutron causing the fission, \( n_2 \) is the resulting fission neutrons which are known as prompt neutrons. They can produce subsequent fissions after thermalization [5]. \( F_1 \) and \( F_2 \) are 2 fission fragments. These fragments can undergo decay producing the so-called delayed neutrons. \( m \) is an integer ranging from 2 to 6 with 2.5 average. Fission chains produce 24 fission fragments with various half life times [6]. This results into a system having 24 states.
To reduce the system order, it is a common practice to group these 24 fragments into 6 groups. Each group contains fragments of comparable half times.

In general, for small and medium size reactors, the effects depending on the space can be neglected and the dynamic behavior can be approximated by the following, so-called, point kinetic equations with six delayed neutron precursor groups:

\[
\dot{n}_i = \frac{\rho_t - \beta}{\Lambda} n_i + \sum_{i=1}^{6} \lambda_i C_{i,t}
\]

(2)

\[
\dot{C}_{i,t} = \frac{\beta_i}{\Lambda} n_t - \lambda_i C_{i,t}, \quad i = 1, \ldots, 6
\]

(3)

where \(n_t\) is the neutron density (\(W\)), \(C_{i,t}\) is the concentration of the \(i\)th group delayed neutron precursor (\(W\)), \(\rho_t\) is the total reactivity, \(\Lambda\) is the effective prompt neutron lifetime (\(s\)), \(\lambda_i\) is the radioactive decay constant of \(i\)th group neutron precursor (\(s^{-1}\)), \(\beta_i\) is the fraction of \(i\)th group delayed neutrons, and \(\beta\) is the total delayed neutron fraction \((\beta = \sum_{i=1}^{6} \beta_i)\). It is important to mention that the six group point kinetic equations (2) and (3) are in fact a set of seven ordinary differential equations; accordingly, their manipulation can result difficult. However, it is still possible to reduce the system order by combining the six precursor groups into an equivalent single group. First, let us define the effective precursor radioactive decay constant \(\lambda\) as

\[
\lambda = \frac{1}{\beta} \sum_{i=1}^{6} \beta_i \lambda_i
\]

(4)

Next, using (4), equations (2) and (3) can be simplified into a second order system given by

\[
\dot{n}_i = \frac{\rho_t - \beta}{\Lambda} n_i + \lambda C_t
\]

(5)

\[
\dot{C}_t = \frac{\beta}{\Lambda} n_t - \lambda C_t
\]

(6)

where \(C_t\) is the equivalent concentration of all delayed neutron precursors. With respect to the total reactivity \(\rho_t\), it has two components, the external reactivity \(\rho_{ext,t}\) and the internal reactivity \(\rho_{int,t}\), that is,

\[
\rho_t = \rho_{ext,t} + \rho_{int,t}
\]

(7)

The external reactivity is related to the position of the control rods. Thus, the external reactivity is considered as the control input of the system. The relationship between the external reactivity and
the rod position can be represented through an empirical static function. On the other hand, the internal reactivity is associated with the effects of the temperature feedback. If the change of the coolant temperature can be neglected, these effects can be described by [7]

\[ \rho_{\text{int},t} = -\alpha Kn_t + \alpha Kn_0 - \gamma \rho_{\text{int},t} \]  

(8)

where \( \alpha \) is the negative reactivity coefficient \( (^\circ \text{C}^{-1}) \) of the fuel due to temperature effects (nuclear Doppler effect), \( K \) is the reciprocal of the reactor heat capacity \( (^\circ \text{C}/(W\cdot s)) \), \( \gamma \) is the reciprocal of mean time for heat transfer to the coolant \( (s^{-1}) \), and \( n_0 \) is the initial neutron density when the external reactivity is equal to zero. This initial neutron density is provided by an external source. Equations (5), (6), and (8) constitute a third order mathematical model of the dynamics of a point nuclear reactor. Defining the state coordinates and the control input as \( x_{1,t} := n_t, x_{2,t} := C_t, x_{3,t} := \rho_{\text{int},t}, u_t := \rho_{\text{ext},t} \), equations (5), (6), and (8) can be represented in the standard state variable form

\[ \begin{align*}
\dot{x}_{1,t} &= -\frac{\beta}{\Lambda} x_{1,t} + \frac{1}{\Lambda} x_{1,t} x_{3,t} + \frac{1}{\Lambda} x_{1,t} u_t \\
\dot{x}_{2,t} &= \frac{\beta}{\Lambda} x_{1,t} - \gamma x_{2,t} \\
\dot{x}_{3,t} &= -\alpha K x_{1,t} + \alpha Kn_0 - \gamma x_{3,t}
\end{align*} \]  

(9)

These equations are the simplest form of representing the nonlinearities, delay and feedback mechanisms inherent in reactor dynamics [8]. The nominal parameters [9] for the model (9) corresponding to TRIGA MARK III research reactor located at ININ, México are showed in Table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.01359875 (^\circ \text{C}^{-1} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>6.433x10^{-2}</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.4024s^{-1}</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>38 ( \mu \text{s} )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.2s^{-1}</td>
</tr>
<tr>
<td>( K )</td>
<td>1/5.21045x10^{4} (^\circ \text{C}/(W\cdot s) )</td>
</tr>
</tbody>
</table>

### 3. LINEAR CASE

As previously mentioned, when the estimation or reconstruction of non-measurable reactor’s states is required, the most common approach found in the literature has been the application of a simple linearization process of the point kinetic equations around a specific equilibrium point. This is justified by the intuitively correct principle that the behavior of the linearized system has
to be acceptably close to the behavior of the nonlinear system within an also acceptable vicinity about the equilibrium point. These results are however somewhat limited and only valid when no large variations occur on the operating conditions. As a first approach on determining the observability of the point kinetic equations in a more general context, a modified linearization procedure is introduced in this section, where any possible equilibrium point is considered. Then, and also in this section, a usual linear observability analysis is performed.

Firstly, a brief summary of essential results on the observability of linear time-invariant systems is presented. Consider the following system

\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]

(10)

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^q \), \( y \in \mathbb{R}^m \), \( A \in \mathbb{R}^{mn} \), \( B \in \mathbb{R}^{nxq} \) and \( C \in \mathbb{R}^{mxn} \)

**Definition 3.1.** The system (10) is said to be observable within the interval time \([t_0, t_1]\), \( t_1 > t_0 \) when the output data \( y(t) \) determine the initial state \( x(t_0) \) completely.

**Theorem 3.1 Kalman Condition for Observability.** System (10) is observable if and only if the observability matrix defined as

\[
\begin{bmatrix}
C \\
CA \\
CA^2 \\
\vdots \\
CA^{n-1}
\end{bmatrix}
\]

is of rank \( n \).

**Remark 3.1** When system (10) is a single output one, that is, \( C \in \mathbb{R}^{1xn} \), the Kalman condition is equivalent to determining if the determinant of the observability matrix is not equal to zero.

Before applying the linearization process to a nonlinear system with respect to an equilibrium point, it is convenient to determine exactly the meaning of this last concept.

**Definition 3.2** Consider a system whose state space representation is given by

\[ \dot{x} = f(x,u) \]

(11)

A value of \( x \) and \( u \) that satisfies

\[ f(x,u) = 0 \]

(12)

is called a system’s equilibrium point.
To determine a general equilibrium point of the nonlinear model of the reactor, system (9) is first set equal to zero

\[
-\frac{\beta}{\Lambda} x_1 + \lambda x_2 + \frac{1}{\Lambda} x_1 x_3 + \frac{1}{\Lambda} x_1 u = 0
\]

(13)

\[
\frac{\beta}{\Lambda} x_1 - \lambda x_2 = 0
\]

(14)

\[
-\alpha K x_1 + \alpha K n_0 - \gamma x_3 = 0
\]

(15)

Then, equations (14) and (15) are expressed in terms of \( x_1 \)

\[
x_2 = \frac{\beta}{\lambda \Lambda} x_1
\]

(16)

\[
x_3 = -\frac{\alpha K}{\gamma} (x_1 - n_0)
\]

(17)

On the other hand, equations (13) and (14) are added term by term to obtain

\[
\frac{1}{\Lambda} x_1 x_3 + \frac{1}{\Lambda} x_1 u = 0
\]

\[
\frac{1}{\Lambda} x_1 (x_3 + u) = 0
\]

(18)

If \( x_1 \neq 0 \), equation (18) can be reduced to

\[
x_1 + u = 0
\]

(19)

By substituting (17) in (19), the following expression is obtained

\[
u = \frac{\alpha K}{\gamma} (x_1 - n_0)
\]

(20)

To specify any constant steady state value for \( x_1 \), the variable \( R \) is used, whose value can be set within the interval

\[
n_0 \leq R \leq R_{\text{max}}
\]

(21)

Thus, the general equilibrium point of the point kinetic equations (9), is denoted by \((\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{u})\), where

\[
\bar{x}_1 = R
\]

(22)

\[
\bar{x}_2 = \frac{\beta}{\lambda \Lambda} R
\]

(23)
The linearization procedure of a three differential equation system with respect to an equilibrium point is now presented. Consider the system

\[
\dot{x}_1 = f_1(x_{1,t}, x_{2,t}, x_{3,t}, u_t), \\
\dot{x}_2 = f_2(x_{1,t}, x_{2,t}, x_{3,t}, u_t), \\
\dot{x}_3 = f_3(x_{1,t}, x_{2,t}, x_{3,t}, u_t)
\]  

Small deviations about the equilibrium \((x_1, x_2, x_3, u)\) are denoted as

\[
w_{1,t} = x_{1,t} - x_1, \\
w_{2,t} = x_{2,t} - x_2, \\
w_{3,t} = x_{3,t} - x_3
\]

Linearization of system (26) w.r.t. the equilibrium \((x_1, x_2, x_3, u)\) in terms of the deviations defined in (27) is carried as follows [10]

\[
\begin{bmatrix}
\dot{w}_{1,t} \\
\dot{w}_{2,t} \\
\dot{w}_{3,t}
\end{bmatrix} = 
\begin{bmatrix}
\frac{\partial f_1(x_1, x_2, x_3, u)}{\partial x_1} & \frac{\partial f_1(x_1, x_2, x_3, u)}{\partial x_2} & \frac{\partial f_1(x_1, x_2, x_3, u)}{\partial x_3} \\
\frac{\partial f_2(x_1, x_2, x_3, u)}{\partial x_1} & \frac{\partial f_2(x_1, x_2, x_3, u)}{\partial x_2} & \frac{\partial f_2(x_1, x_2, x_3, u)}{\partial x_3} \\
\frac{\partial f_3(x_1, x_2, x_3, u)}{\partial x_1} & \frac{\partial f_3(x_1, x_2, x_3, u)}{\partial x_2} & \frac{\partial f_3(x_1, x_2, x_3, u)}{\partial x_3}
\end{bmatrix}
\begin{bmatrix}
w_{1,t} \\
w_{2,t} \\
w_{3,t}
\end{bmatrix} + 
\begin{bmatrix}
\frac{\partial f_1(x_1, x_2, x_3, u)}{\partial u} \\
\frac{\partial f_2(x_1, x_2, x_3, u)}{\partial u} \\
\frac{\partial f_3(x_1, x_2, x_3, u)}{\partial u}
\end{bmatrix} v
\]  

The result of applying (28) to system (9) with respect to the general equilibrium point (22-25) is

\[
\begin{bmatrix}
\dot{w}_{1,t} \\
\dot{w}_{2,t} \\
\dot{w}_{3,t}
\end{bmatrix} = 
\begin{bmatrix}
-\beta & \lambda & \frac{R}{\Lambda} \\
\beta & -\lambda & 0 \\
-\alpha K & 0 & -\gamma
\end{bmatrix}
\begin{bmatrix}
w_{1,t} \\
w_{2,t} \\
w_{3,t}
\end{bmatrix} + 
\begin{bmatrix}
\frac{R}{\Lambda} \\
0 \\
0
\end{bmatrix} v
\]  

Thus,
Since in general the only measurable variable of the point kinetic equations is the neutron density, the output of system (9) is defined as
\[ y_t := x_{1,t} \].
For system (29) with state vector \( w_t \), the output is
\[ \tau_t y_t = C w_t \]
where
\[ C := \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \] and \( \tau_t \in \mathbb{R} \).
At this point, all the elements to form the observability matrix are available, which, using theorem 3.1, can be expressed as
\[
\Theta = \begin{bmatrix}
1 & 0 & 0 \\
-\frac{\beta}{\Lambda} & \lambda & \frac{R}{\Lambda} \\
\frac{\beta^2}{\Lambda^2} + \frac{\lambda R}{\Lambda} & \frac{R}{\Lambda} & \frac{\lambda}{\Lambda} \\
\end{bmatrix}
\]
Finally, the determinant of the observability matrix is given by
\[
\det(\Theta) = -\frac{B\lambda R}{\Lambda^2} - \frac{\gamma R}{\Lambda} + \frac{\lambda R}{\Lambda^2} + \frac{\lambda^2 R}{\Lambda} = \left(\frac{\lambda^2 - \lambda \gamma}{\Lambda}\right) R = \frac{\lambda (\lambda - \gamma) R}{\Lambda}
\]
From this last result, it can be concluded that two conditions have to be met to preserve the observability of the system: 1) \( R > 0 \), and 2) \( \lambda - \gamma \neq 0 \). So, for example, for Mexico TRIGA reactor the first condition is always satisfied since \( n_0 \leq R \leq R_{\text{max}} \), and always \( n_0 > 0 \). With respect to the second condition, from the nominal values of the parameters given in table I, it can be verified that effectively \( \lambda \neq \gamma \). Thus, the corresponding linearized model for Mexico TRIGA reactor is observable.

### 4. NONLINEAR CASE

From the analysis presented in the previous section, it could be concluded that the point kinetic equations (9) are, in general, always observable. Even considering that this assertion is partially correct, the generalization must be carefully carried out since, as it will be noticed, the observability conditions for the nonlinear case may result more restrictive. Indeed, and contrary to the linear case, the observability of the nonlinear case is a characteristic that depends on the system’s control signal values.

Consider the nonlinear system described by
\[ \dot{x}_t = f(x_t, u_t, t), \quad y_t =Cx_t \] 

(30)

where \( x_t \in \mathbb{R}^n \) is the system state at time \( t \geq 0 \), \( y_t \in \mathbb{R} \) is the system output, \( u_t \in \mathbb{R} \) is the control action, \( C \in \mathbb{R}^{1 \times n} \) is an output matrix and \( f : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}^n \). Note: For the sake of briefness, system (30) is SISO but results presented in this work can be easily extended to MIMO systems.

Let us define the extended output vector as

\[ Y_t := \begin{bmatrix} y_t & \dot{y}_t & \cdots & y_t^{(n-1)} \end{bmatrix}^T \] 

(31)

and the observability matrix for the nonlinear case as

\[ Q = \frac{\partial Y_t}{\partial x_t} \] 

(32)

Results on the observability of system (30) are presented in [11] and references there. In this work, we summarize these results by means of the following corollary:

**Corollary 4.1** System (30) is **locally observable** in a neighborhood of the point \( x_t \) at time \( t \), if

\[ \det Q \neq 0 \] 

(33)

Although this condition is very alike to the Kalman condition for linear systems, it can produce different results as it will be soon seen. First, the Kalman condition is necessary and sufficient whereas (33) is only sufficient. Let us begin with the analysis computing the corresponding extended output vector for system (9)

\[ Y_t = \begin{bmatrix} y_t \\
\dot{y}_t \\
\ddot{y}_t \\
\dddot{y}_t \\
\dddot{x}_t \\
\dddot{x}_t \\
\dddot{x}_t \end{bmatrix} = \begin{bmatrix} x_{1,t} \\
\dot{x}_{1,t} \\
\ddot{x}_{1,t} \\
\dddot{x}_{1,t} \end{bmatrix} \] 

(34)

Determining \( \dot{x}_{1,t} \) is straightforward; however, computing \( \ddot{x}_{1,t} \) is laborious. The result is
\[
\dot{x}_{1,t} = \frac{\beta^2}{\Lambda} x_{1,t} - \frac{\beta\lambda}{\Lambda} x_{2,t} - \frac{\beta}{\Lambda^2} x_{1,t} x_{3,t} - \frac{\beta\lambda}{\Lambda} x_{1,t} - \frac{\lambda^2}{\Lambda} x_{2,t} - \frac{\beta}{\Lambda^2} x_{1,t} x_{3,t} + \frac{\lambda}{\Lambda} x_{1,t} x_{3,t} + \frac{1}{\Lambda^2} x_{1,t} x_{3,t} u_t - \frac{\alpha K}{\Lambda} x_{1,t}^2 + \frac{\alpha K_n}{\Lambda} x_{1,t} - \frac{\gamma}{\Lambda} x_{1,t} x_{3,t} - \frac{\beta}{\Lambda^2} x_{1,t} u_t + \frac{\lambda}{\Lambda} x_{2,t} u_t + \frac{1}{\Lambda^2} x_{1,t} x_{3,t} u_t + \frac{1}{\Lambda^2} x_{1,t} x_{3,t} u_t + \frac{1}{\Lambda^2} x_{1,t} x_{3,t} u_t
\]

After grouping terms,

\[
\dot{x}_{1,t} = \left(\frac{\beta^2}{\Lambda} + \frac{\beta\lambda}{\Lambda} + \frac{\alpha K_n}{\Lambda}\right) x_{1,t} - \frac{\alpha K}{\Lambda} x_{1,t}^2 - \left(\frac{2\beta}{\Lambda^2} + \frac{\gamma}{\Lambda}\right) x_{1,t} x_{3,t} + \frac{1}{\Lambda^2} x_{1,t} x_{3,t} u_t - \frac{2\beta}{\Lambda^2} x_{1,t} u_t + \frac{1}{\Lambda^2} x_{1,t} u_t^2 + \frac{1}{\Lambda^2} x_{1,t} u_t^2
\]

The observability matrix is now given by

\[
Q = \frac{\partial Y_t}{\partial x_t} = \begin{bmatrix}
1 & 0 & 0 \\
- \beta + \frac{1}{\Lambda} x_{3,t} + \frac{1}{\Lambda} u_t & \lambda & \frac{1}{\Lambda} x_{1,t} \\
\frac{\partial \ddot{x}_{1,t}}{\partial x_{1,t}} & \frac{\partial \ddot{x}_{1,t}}{\partial x_{2,t}} & \frac{\partial \ddot{x}_{1,t}}{\partial x_{3,t}}
\end{bmatrix}
\]

where

\[
\frac{\partial \ddot{x}_{1,t}}{\partial x_{1,t}} = -\left(\frac{\beta\lambda}{\Lambda} + \lambda^2\right) + \frac{\lambda}{\Lambda} x_{3,t} + \frac{\lambda}{\Lambda} u_t
\]

\[
\frac{\partial \ddot{x}_{1,t}}{\partial x_{2,t}} = -\left(\frac{2\beta}{\Lambda^2} + \frac{\gamma}{\Lambda}\right) x_{1,t} + \frac{2}{\Lambda^2} x_{1,t} x_{3,t} + \frac{2}{\Lambda^2} x_{1,t} u_t + \frac{\lambda}{\Lambda} x_{2,t}
\]

The determinant of matrix (35) is given by

\[
\det(Q) = \lambda \left\{-\left(\frac{2\beta}{\Lambda^2} + \frac{\gamma}{\Lambda}\right) x_{1,t} + \frac{2}{\Lambda^2} x_{1,t} x_{3,t} + \frac{2}{\Lambda^2} x_{1,t} u_t + \frac{\lambda}{\Lambda} x_{2,t}\right\} - \frac{1}{\Lambda} x_{1,t} \left\{-\left(\frac{\beta\lambda}{\Lambda} + \lambda^2\right) + \frac{\lambda}{\Lambda} x_{3,t} + \frac{\lambda}{\Lambda} u_t\right\}
\]

\[
= \left(\frac{\lambda^2}{\Lambda} - \frac{\beta\lambda}{\Lambda} - \frac{\gamma\lambda}{\Lambda}\right) x_{1,t} + \frac{\lambda}{\Lambda^2} x_{1,t} x_{3,t} + \frac{\lambda}{\Lambda^2} x_{1,t} u_t + \frac{\lambda^2}{\Lambda^2} x_{2,t}
\]

According to corollary 1, the point kinetic equations (9) are always observable whenever \(\det(Q) \neq 0\). Consequently, \(\det(Q) = 0\) defines a “dangerous” manifold, which should be avoided. This leads to the following restriction on the control signal.
\[ u_i \neq \beta + \gamma \lambda - \lambda x_{3,i} - \lambda \frac{x_{2,i}}{x_{1,i}} \]  

This restriction must be always satisfied to guarantee the observability of system (9).

5. CONCLUSIONS

The key issue of existence of a solution to the problem of reconstructing the non-measurable states of a nuclear reactor using the observable states and the system input has been considered in this work. According to the analysis presented, an important conclusion is that the observability result based on a linearization with respect to a particular equilibrium point is not sufficiently reliable when the operating conditions vary considerably. It should be understood that even though the concepts of observability for linear and nonlinear systems are very alike, in practice, the observability conditions for each case may differ noticeably from each other.

REFERENCES